

Skills Needed for Mathematical Problem Solving

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Why Is Mathematical Problem Solving Important?

It helps students

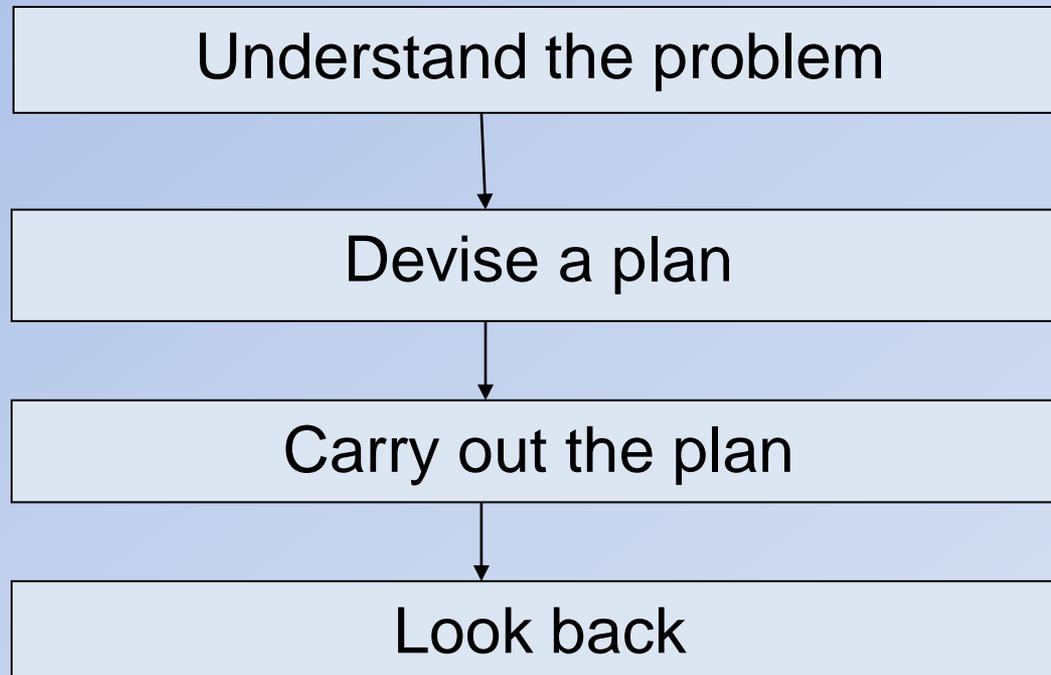
1. develop and improve the generic ability to solve real life problems,
2. develop critical thinking skills and reasoning,
3. gain deep understanding of concepts,
4. work in groups, interact with and help each other.

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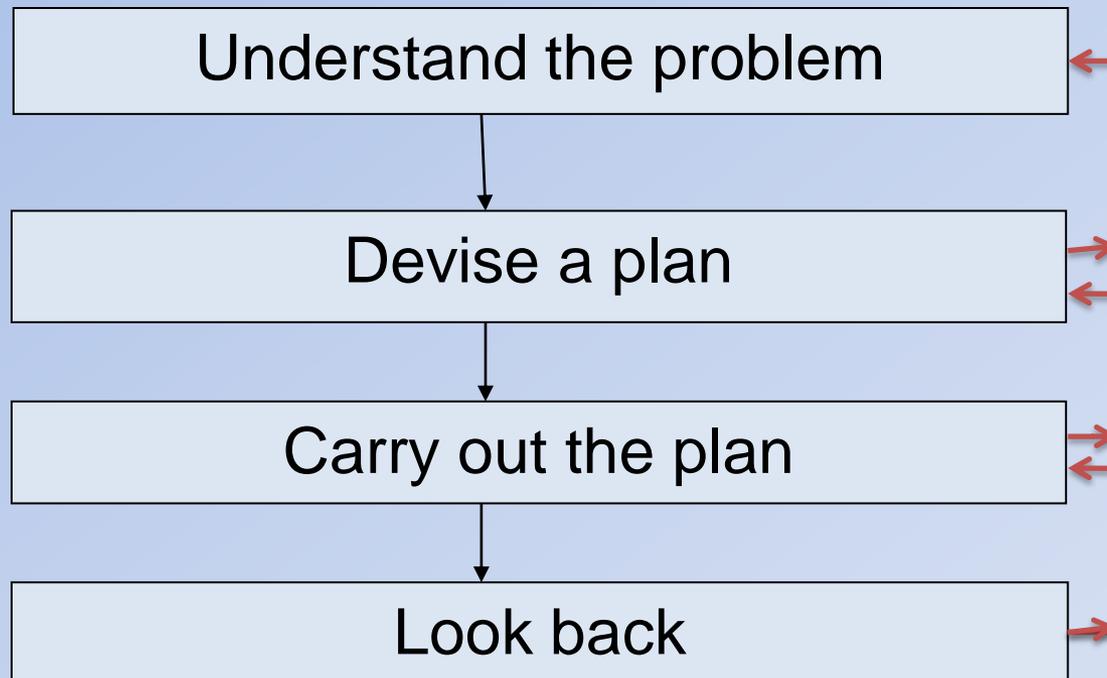
1 - Why do students find mathematical Problem solving difficult to learn?

2- Why do instructors find mathematical problem solving difficult to teach?

Mathematical Problem Solving as a Linear Process



Mathematical Problem Solving as a Non Linear Process



Factors and Skills Involved in Problem Solving

1. **Concepts and facts:** arithmetic, algebraic, geometric, statistical ...
2. **Skills:** arithmetic, algebraic geometric manipulations, estimation, approximation, reading with understanding ...
3. **Thinking and Reasoning:** Inductive and deductive reasoning, critical and creative thinking, use of heuristics ...
4. **Metacognition:** Analyze and control one's thinking.
5. **Group Work:** Work in groups to overcome difficulties solving challenging problems , ...
6. **Attitudes:** Persevere, have self confidence, appreciate the power of problem solving ...

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Basic Skills in Problem Solving

Reading with understanding

- It is the first step in problem solving and students cannot make any progress if the problem is not understood.
- Our students have difficulties in reading with understanding and extracting the information from the text of the problem.
- This skill has to be taught explicitly to our students: underlining key words, extracting information from the text,...

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Concepts, Facts and Problem Solving

I will use examples of problem solving activities I facilitated in my ADM classes.

Example 8, page 354.

It takes Carla 1 hour longer to mow the lawn than it takes Sharon to mow the lawn. If they can mow the lawn in 5 hours working together, then how long would it take each girl by herself?

Concept involved: Rate of Work

Method used

- A. I explained the concept of rate.
- B. I helped students solve the problem.

Homework (Solved at home and finished in class)

- John takes 3 hours longer than Andrew to peel 500 pounds (lb) of apples. If together they can peel 500 lb of apples in 8 hours, then how long would it take each one working alone?

Exercise 85, page 357

Concept involved: Rate of work

Method used

- A. I checked students' works and found out that the great majority did not know what to do with the 500. I concluded that they may not have understood the concept of rate of work.
- B. I explained the rate of work once more.
- C. I helped students solve the problem.

Homework (Solved at home and finished in class)

- It takes pump A 2 hours less time than pump B to empty a certain swimming pool. Pump A is started at 8:00 A.M., and pump B is started at 11:00 A.M. If the pool is still half full at 5:00 P.M., then how long would it take pump A working alone?

Exercise 89, page 358

Concept involved: Rate of work

Method used

A. I checked students' works and found out that they encountered two major difficulties: which time to use in the main formula and what is the total work when the two pumps work together. Very few students managed to formulate and solve the problem.

B. I explained the concept of rate of work with more examples related to problems of speed, time and distance.

C. I helped students solve the problem

QUIZ 1 - Fall 2007 – V2

- It takes pump B 2 hours more time than pump A to fill a swimming pool. Both pumps are started at 7 am. At 10 am pump A breaks down and it took 1 hour to repair it and then was restarted again. At 3 pm 80 % of the swimming pool was filled with water. How long would it take each pump working alone to fill the swimming pool?
- About half of the students managed to solve the problem completely and a quarter formulated the problem with one or two minor mistakes.

How Can We Teach A Mathematical Concept Using Problem Solving?

Example: Population Problem to Introduce Exponential Functions

- The present population of the UAE is 4.5 million. If we assume that the population grows at a an annual rate $r = 3\%$ for the next 15 years, what will be the population in t years?

Method of Teaching/Facilitating

- Review percent
- Do the first step(s) for the students
 1. For $t = 1$ year,
$$P(1) = 4.5 + 3\% 4.5$$
$$= 4.5 (1 + 3\%)$$
 2. For $t = 2$ years,
$$P(2) = P1 + 3\% P1$$
$$= 4.5 (1 + 3\%) + 3\% 4.5 (1 + 3\%)$$
$$= 4.5 (1+3\%) (1+3\%)$$
$$= 4.5 (1+3\%)^2$$
- Help students find a formula
 - $P(t) = 4.5 (1+3\%)^t$
- Explain that this is a new function with the form $P(t) = k a^t$ and is called an exponential function.

Thinking and Reasoning (homework over a long period)

- Problem: Two boats on opposite banks of a river start moving towards each other. They first pass each other 1400 meters from one bank. They each continue to the opposite bank, immediately turn around and start back to the other bank. When they pass each other a second time, they are 600 meters from the other bank. We assume that each boat travels at a constant speed all along the journey. Is it possible to find the width of the river using the given information?

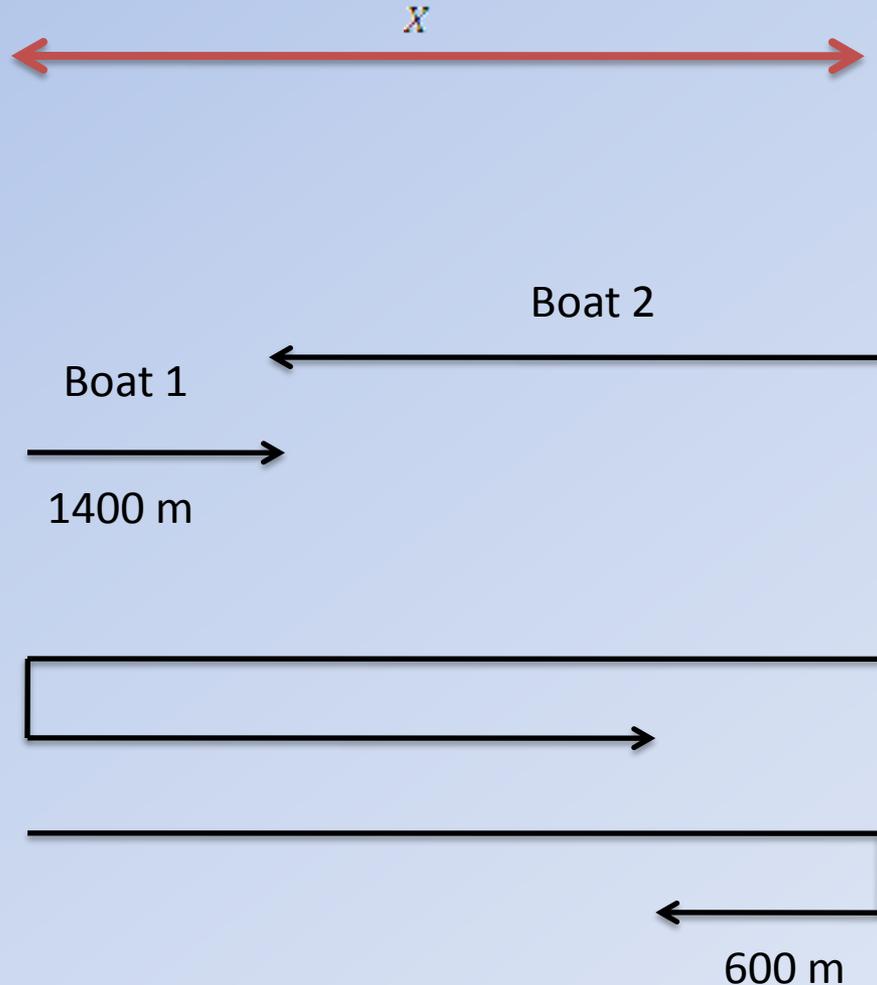
Can the problem be solved?

- Boat (1): speed S_1
- Boat (2): speed S_2

- $S_1 t_1 = 1400$
- $1400 + S_2 t_1 = x$

- $S_1 t_2 = X + 600$

- $S_2 t_2 = 2x - 600$



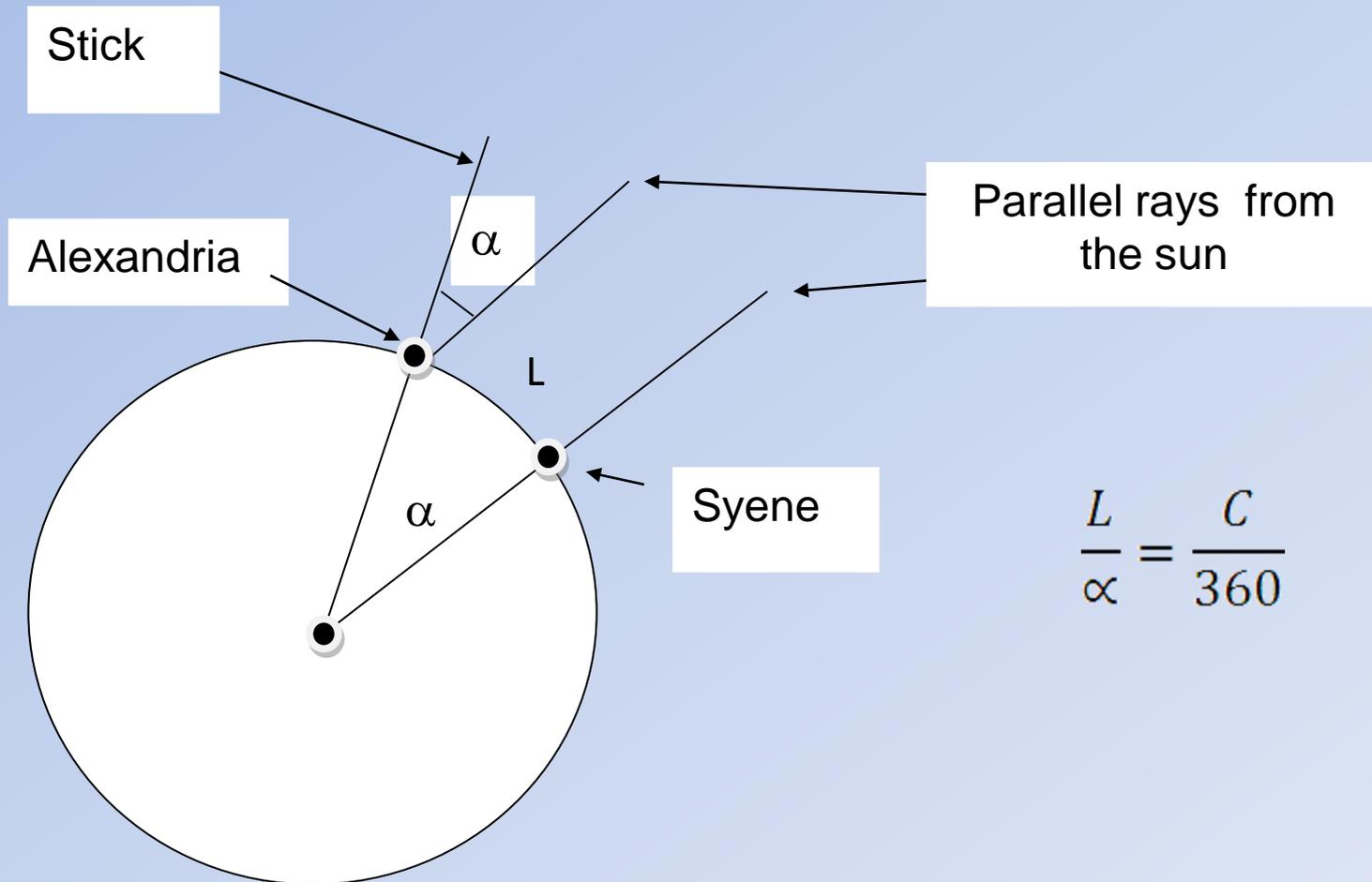
Open Ended Problems

Problem: Create a set of data points that satisfies the following conditions:

- The set includes 8 data values.
- The range of the data set is 20.
- The median is equal to the mean.
- Show that your data set satisfies the conditions.

Attitude Towards Problem Solving

Eratosthenes of Cyrene (276 BC-194 BC)



Metacognition and Problem Solving

- Metacognitive skills have to be taught explicitly.
- Only genuine mathematical problems, that students have not solved before, help them develop metacognitive skills.
- Students need to explain to other students and the teacher their way of thinking.
- I sometimes use examples to explain my own thinking in solving problems.

Metacognition: Example

- Problem : Ahmed walked at a constant speed of 6 km/hour along a straight line from A to B, then walked back along the same line from B to A at a constant speed of 4 km/hour. What is the average speed over the entire trip?
(Definition: Average speed = total distance / total time)

Group Work

- When the problem given to students is challenging, students understand the need to work in groups.
- It was shown that cooperative learning and metacognitive activities have positive effects on the students' abilities to solve problems.
- Group work also prepares students for the future where they have to work together on large problems and projects.

Conclusion

- Many skills and factors are involved when genuine mathematical problems are being solved.
- Instructors have to understand and be familiar with these factors and skills.
- Instructors also need to design activities and guide students to develop and use these skills.
- Students must be aware of these factors and skills.
- It is possible to design problems that focus on a limited number of skills and factors.
- Students develop these skills mainly when genuine mathematical problems solving is taking place.